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Heat and mass transfer induced by natural convection combined with thermo-diffusion within a horizontal porous enclosure heated and salted from its shorts sides

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Abstract : Fluid flow and Heat and mass transfers induced by natural convection combined with thermo-diffusion phenomenon "Soret effect" within a shallow porous enclosure submitted to lateral uniform heat and mass fluxes is studied in this work. The interest is mainly centered on a specific situation where the buoyancy forces ratio N and the Soret parameter S_P are such that $N = -1/(1 - S_P)$. In the absence of the Soret effect, this case corresponds to $N = -1$ for which the buoyancy forces induced by thermal and solutal effects are opposing and of equal intensity. An analytical solution is derived on the basis of the parallel flow approximation, and is subsequently validated numerically by solving the complete governing equations using a finite difference method. The effect of the parameters governing the problem on fluid flow properties and heat and mass transfer characteristics is analyzed. The existence of multiplicity of solutions is also discussed.

Key words : Heat and mass transfers, Soret effect, porous medium, parallel flow assumption, subcritical convection.

1. Introduction

The Soret effect on double diffusive natural convection developed in a fluid-saturated porous media has received a growing attention as it is encountered in many natural, environmental and engineering processes. The water movements in geothermal reservoirs, the diffusion of the radioactive substances in the underground deposits and the diffusion of the chemical elements in the porous reactive beds are some applications among others where this type of problems can be observed. From a theoretical point of view, this interest is justified by the existence of specific behaviours (multiplicity of solutions, hysteresis phenomenon, Hopf's bifurcations, etc.) attributed to the thermo-diffusion phenomenon. Most of the experimental studies on the thermo-diffusion were dedicated to the measurement of Soret coefficient for various mixtures using different techniques [1-3]. For binary mixtures, the Soret coefficient is measured as the ratio of the thermo-diffusion coefficient to the molecular diffusion and the accuracy of the measurements is inevitably influenced by convection. In the review by Platten [4], relating the different techniques used to measure the Soret coefficient, the reader learns that each technique has its own limitation, which means that the experimental approach of the phenomenon remains still a real challenge for the experimenters. The literature review shows also that some experiments were dedicated to measure bifurcations phenomena in a porous layer [5] or to measure the separation of species [6-7]. The theoretical efforts on the subject were dedicated to the bifurcation phenomena [8-9], multiplicity of solutions [10-11], separation optimization [12], etc.

The main objective of the present investigation consists to study analytically and numerically the Soret effect on thermosolutal natural convection induced in a horizontal Darcy porous layer subject to lateral heat and mass fluxes. This problem is classified in the category of problems where heat and mass gradients are imposed horizontally. In the absence of Soret effect, an equilibrium solution is possible when thermal and solutal buoyancy forces are opposing each other. In this investigation, the attention is mainly focused on the particular situation where $N = -1/(1 - S_P)$. An appropriate analytical solution is derived using the parallel flow approximation. The features predicted by the analytical analysis are confirmed numerically for a wide range of the governing parameters and interesting flow bifurcations phenomena are discussed.

2. Problem formulation

The studied configuration is sketched in Fig. 1. It consists of an isotropic, homogeneous and saturated horizontal Darcy porous layer of height H' and width L' such that $A_r = L'/H' \gg 1$. The lateral walls of the enclosure are subjected to uniform fluxes of heat q' and mass j' while its long horizontal walls are considered adiabatic and impermeable to mass transfer. The diluted binary solution that saturates the porous medium is modeled as a Boussinesq incompressible fluid for which the fluid density varies according to the relationship given by:

$$\rho = \rho_0[1 - \beta_T(T' - T'_0) - \beta_S(S' - S'_0] \tag{1}$$

The subscript "0" refers to conditions at the origin of the coordinates system taken in the geometric centre of the cavity.

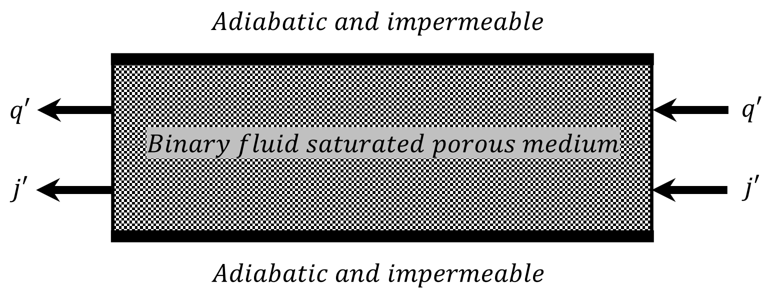

Figure 1: Shematic of the physical problem

Assuming constant physical properties and using the Boussinesq approximation, the dimensionless governing equations obeying the Darcy model are as follows:

$$\eta \frac{\partial \xi}{\partial t} + \xi = R_T \left(\frac{\partial T}{\partial x} + N \frac{\partial S}{\partial x} \right) \tag{2}$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \nabla^2 T \tag{3}$$

$$\varepsilon \frac{\partial S}{\partial t} + u \frac{\partial S}{\partial x} + v \frac{\partial S}{\partial y} = \frac{1}{Le} (\nabla^2 S + S_P \nabla^2 T) \tag{4}$$

$$\nabla^2 \psi = -\xi \tag{5}$$

$$u = \frac{\partial \psi}{\partial y} \quad ; \quad v = -\frac{\partial \psi}{\partial x} \tag{6}$$

The boundary conditions associated to this problem are such that:

$$\begin{cases} x = \pm A_r/2 \ : \ \psi = 0 \ , \ \dfrac{\partial T}{\partial x} = 1 \ , \ \dfrac{\partial S}{\partial x} = 1 - S_P \\ y = \pm 1/2 \ : \ \psi = 0 \ , \ \dfrac{\partial T}{\partial y} = 0 \ , \ \dfrac{\partial S}{\partial y} = 0 \end{cases} \tag{7}$$

where ξ, ψ, T, S, u and v are the dimensionless vorticity, stream function, temperature, concentration, horizontal and vertical components of the velocity, respectively.
In the governing equations, dimensionless parameters appear explicitly which are the thermal Rayleigh number, R_T; the Lewis number, Le; the Soret parameter, S_P; and the solutal to thermal buoyancy ratio, N. They describe respectively the thermal driving force, the relative importance of the thermal diffusivity with respect to the solute one, the thermo-diffusion phenomenon (Soret effect) and the importance of solutal buoyancy forces due to the applied mass flux, j'.

The Nusselt and Sherwood numbers, which characterize the heat and mass transfer rates, are respectively given by:

$$\overline{Nu} = \int_{-1/2}^{1/2} Nu(y)dy \qquad \text{and} \qquad \overline{Sh} = \int_{-1/2}^{1/2} Sh(y)dy \tag{8}$$

where:

$$\begin{cases} Nu(y) = \lim\limits_{\delta x \to 0} \delta x/\delta T(y) \ = \lim\limits_{\delta x \to 0} 1/(\delta T(y)/\delta x) = 1/(\partial T/\partial x)_{x=0} \\ Sh(y) = \lim\limits_{\delta x \to 0} \delta x/\delta S(y) \ = \lim\limits_{\delta x \to 0} 1/(\delta S(y)/\delta x) = 1/(\partial S/\partial x)_{x=0} \end{cases}$$

3. Results and discussion

In general, it is not possible to perform an exact analytical solution for the governing equations. However, in the case of shallow enclosures ($A_r \gg 1$), an approximate analytical solution can be developed in the central part of the cavity. This solution is based on the parallel flow approximation which allows the following simplifications:

$$\psi(x,y) \cong \psi(y) \ \ , \ T(x,y) \cong C_T x + \theta_T(y) \ \text{ and } \ S(x,y) \cong C_S x + \theta_S(y)$$

where C_T and C_s are respectively unknown constant temperature and concentration gradients in the horizontal direction.

Taking these approximations into account, a set of ordinary differential equations is obtained. The analytical resolution of these equations leads to analytical expressions of ψ, T and S:

$$\psi(y) = \psi_0(-4y^2 + 1) \tag{9}$$

$$T(x,y) = C_T x + \psi_0 C_T \left(\frac{-4}{3} y^3 + y\right) \tag{10}$$

$$S(x,y) = C_S x + \psi_0 (C_S Le - C_T S_P) \left(\frac{-4}{3} y^3 + y\right) \tag{11}$$

Where ψ_0 is the stream function at the centre of the enclosure; it is defined by the following expression:

$$\psi_0 = \frac{R_T}{8}(C_T + NC_S) \tag{12}$$

By performing global balances of energy and solute transfers across any transversal section of the enclosure, we obtain the constant temperature and concentration horizontal gradients C_T and C_S:

$$C_T = \frac{1}{1 + 8{\psi_0}^2/15} \tag{13}$$

$$C_S = \frac{1}{1 + 8Le^2{\psi_0}^2/15} - S_P \frac{(1 - 8Le{\psi_0}^2/15)}{(1 + 8Le^2{\psi_0}^2/15)(1 + 8{\psi_0}^2/15)} \tag{14}$$

The local Nusselt and Sherwood numbers are found to be constant, they are given by:

$$Nu = \overline{Nu} = \frac{1}{C_T} = 1 + 8{\psi_0}^2/15 \tag{15}$$

$$Sh = \overline{Sh} = \frac{1}{C_S} = \frac{(1 + 8Le^2{\psi_0}^2/15)(1 + 8{\psi_0}^2/15)}{(1 + 8{\psi_0}^2/15) - S_P(1 - 8Le{\psi_0}^2/15)} \tag{16}$$

By combining the equations (12), (13) and (14), an equation for ψ_0 is established:

$$A{\psi_0}^5 + B{\psi_0}^3 - C{\psi_0}^2 + D\psi_0 - E = 0 \tag{17}$$

where A, B, C, D and E are expressed in terms of the governing parameters of the problem as follows:

$$\begin{cases} A = \ 512Le^2 \\ B = 960(Le^2 + 1) \\ C = 120R_T(Le^2 + NS_P Le + N) \\ D = 1800 \\ E = 225R_T(1 - NS_P + N) \end{cases}$$

Mathematical analysis of the equation (17), with $N = -1/(1 - S_P)$ corresponding to $E = 0$, shows that this equation has only two solutions (when they exist). At sufficiently large value of R_T, these solutions are defined by $\psi_0 \propto {R_T}^{1/3}$ and $\psi_0 \propto {R_T}^{-1}$ and the corresponding flows rotate in the same direction.

Also, it can be easily demonstrated that the $S_P - Le$ plane can be divided into two regions I and II delineated in Fig. 2 and characterized by counter-clockwise and clockwise flows, respectively. The thermodiffusion effect on the flow rotation is clearly shown in this figure.

In other hand, it appears from the equation (17) that no super-critical bifurcation is possible for the parallel flow solution; only a sub-critical one exists and the latter occurs at $R_T = R_T^{sub}$ given by:

$$R_T^{sub} = \frac{\tilde{\psi}_0 (4A{\tilde{\psi}_0}^2 + 2B)}{120|Le^2 + (S_P Le + 1)/(S_P - 1)|} \tag{18}$$

where $\tilde{\psi}_0 = \sqrt{(-B + \sqrt{\Delta})/6A}$ with $\Delta = B^2 + 12AD$

The Soret effect on the critical Rayleigh number, characterizing the onset of the sub-critical natural convection, is shown in Fig. 3. The variations of R_T^{sub} with S_P are illustrated for different values of Le ($Le = 3$ and $Le = 10$). It can be seen from this figure that the Soret effect can play a stabilizing ($S_P < 1 - 1/Le$ and $S_P > 1$) or a destabilizing ($1 - 1/Le < S_P < 1$) role whatever the considered binary mixture (value of Le) is.

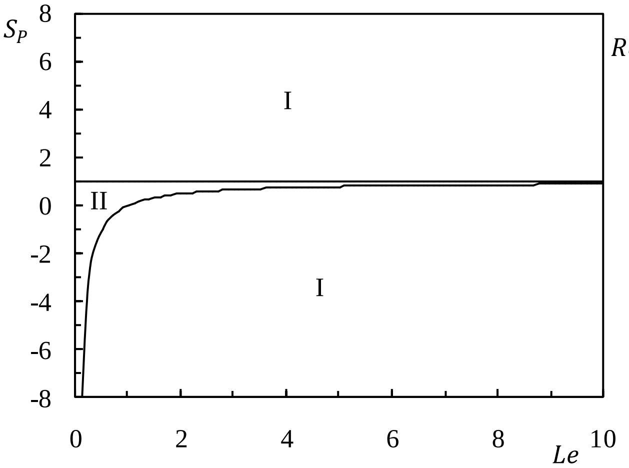

Fig. 2: Delimitation of regions in the ($S_P - Le$) plane according to the flow direction

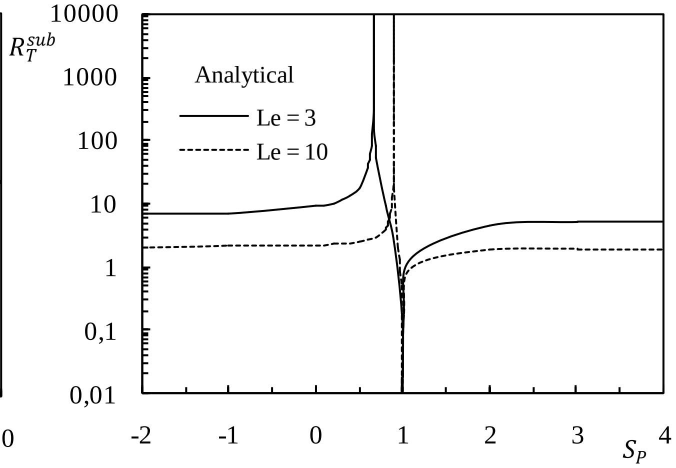

Fig. 3: Variations of R_T^{sub} with S_P for different values of Le

The effect of the thermal Rayleigh number, R_T, on the fluid flow and heat transfer characteristics is illustrated in Fig. 4 in terms of ψ_0 (4-a) and Nu (4-b) variations with R_T for different combinations (Le, S_P). The combinations of Le and S_P are chosen so that both clockwise and counter clockwise flows can be observed.

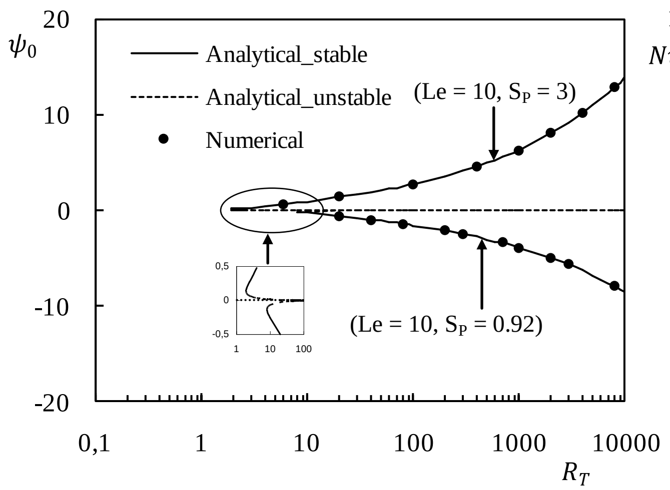

Fig. 4-a: Effect of R_T on ψ_0 for different combinations (Le, S_P)

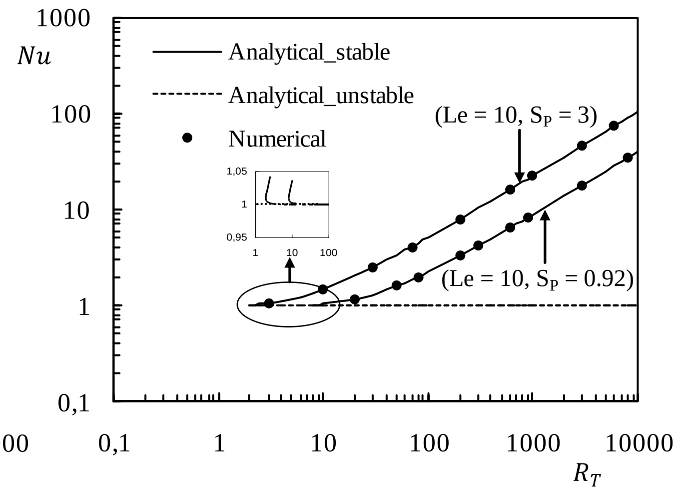

Fig. 4-b: Effect of R_T on Nu for different combinations (Le, S_P)

As shown in Fig. 4, the onset of the subcritical convection occurs at the critical Rayleigh number $R_T^{sub} = 1.915$ and 8.045 respectively for $(Le, S_P) = (10{,}3)$ and (10, 0.92). Therefore, $R_T^{sub}\,(S_P = 0.92) > R_T^{sub}\,(S_P = 3)$, which is in accordance with the results of Fig. 3 illustrating the variations of R_T^{sub} with S_P. It can be seen from Fig. 4 that only one of the two analytical solutions is validated numerically; it is termed as a "stable" solution. The other solution could not be validated numerically and it is termed as "unstable". Figs. 4(a) and 4(b) show that $|\psi_0|$ and Nu corresponding to the stable branches increase with R_T. Analytically, Nu varies as ${R_T}^{2/3}$ at large R_T. For the unstable branches, these quantities are nearly constant and close to 0 and 1 respectively (i.e. values of the purely diffusive regime).

4. Conclusion

Fluid flows and heat and mass transfers induced by natural convection combined with Soret effect within a horizontal porous layer submitted to lateral uniform heat and mass fluxes is studied. The attention was focused on the particular situation for which the rest state is a solution of the problem. An excellent agreement between the stable analytical results, based on the parallel flow approximation, and those numerical is observed. Only the sub-critical bifurcation was found possible for the parallel flows for this case and its threshold was determined analytically versus the governing parameters.

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