



## Numerical Study of Natural Convection Heat and Mass Transfer through a Saturated Porous Medium in Horizontal Cylindrical Annulus

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**Abstract:** Two-dimensional heat and mass transfer of natural convection in an annular cylindrical space filled with fluid-saturated porous medium, is analyzed by solving numerically the mass balance, momentum, energy and concentration equations, using Darcy's law and Boussinesq approximation. Both walls delimiting the annular space are maintained at two uniform different temperatures and concentrations. The external parameter considered is Rayleigh-Darcy number. For the present work, the heat and mass transfer for natural convection is studied for the case of aiding equal buoyancies, where the flow is generated in a cooperative mode by both temperature and solutal gradients. The local Nusselt and Sherwood numbers are presented in term of the external parameter.

**Keywords:** Heat and mass transfer, natural convection, porous media, cylindrical annulus.

### 1. Introduction

The heat and mass transfer of natural convection was a subject of many theoretical, numerical and experimental studies that have dealt with heat and mass transfer confined into different vertical and horizontal annular enclosures. The study of heat and mass transfer in annular spaces is of fundamental importance; that is because it is often met in many practical applications. These annular spaces have different geometries and can be partly or completely filled with porous media. Interest in the phenomena of heat and mass transfer by natural convection is due to many potential applications in the engineering processes which involve the chemical and oil and gas industries, thermal recovery process, the underground spreading of chemical waste and other pollutants, evaporation, cooling and solidification are few other application areas where combined thermo-solutal convection in porous media can be observed.

F.M. Mahfouz [1] has investigated a buoyancy driven flow and associated heat convection in an elliptical enclosure. The enclosure which is the space between two horizontal concentric confocal elliptic tubes is heated through its inner tube surface which is maintained at either uniform temperature or uniform heat flux. N. Allouache and al. [2] analyzed a solid adsorption refrigerator using activated carbon/methanol pair. It is a contribution to technology development of solar cooling systems. The main objective consists to analyze the heat and mass transfer in an annular porous adsorber that is the most important component of the system. The porous medium is contained in the annular space and the adsorber is heated by solar energy. A general model equation is used for modelling the transient heat and mass transfer. Khanafer and al. [3] studied a numerical investigation of natural convection heat transfer within a two-dimensional, horizontal annulus that is partially filled with a fluid-saturated porous medium. In addition, the porous sleeve is considered to be press fitted to the inner surface of the outer cylinder. Kumari and Nath [4] studied the unsteady natural convection flow from a horizontal cylindrical annulus filled with a non-Darcy porous medium. The unsteadiness in the problem arises due to the impulsive change in the wall temperature of the outer cylinder. The Navier-Stokes equations along with the energy equation governing the unsteady natural convection flow have been solved by the finite-volume method. Ramadan Y. Sakr and al. [5] presented experimental and numerical studies for natural convection in two dimensional region formed by constant flux heat horizontal elliptic tube concentrically located in a larger, isothermally cooled horizontal cylinder were investigated. The effects of the orientation angle as well as other parameters such as elliptic cylinder axis ratio and hydraulic radius ratio on the flow and heat transfer characteristics are investigated numerically. Yong Shi and al. [6] presented a finite difference-based lattice BGK model for thermal flows is proposed based on the double-distribution function approach; they applied this model to simulate natural convection heat transfer in a horizontal concentric annulus bounded by two stationary cylinders with different temperatures. Edimilson J and al. [7] examined a numerical computation for laminar and

turbulent natural convection within a horizontal cylindrical annulus filled with a fluid saturated porous medium. Computations covered the range of  $25 < Ra_m < 500$  and  $3.2 \times 10^{-4} > Da > 3.2 \times 10^{-6}$  and made use of the finite volume method. The macroscopic k- $\epsilon$  turbulence model with wall function is used to handle turbulent flows in porous media. Leong and Lai [8] presented a natural convection in concentric cylinders with a porous sleeve, analytical solutions obtained through perturbation method and Fourier transform. The porous sleeve is press-fitted to the inner surface of the outer cylinder. Both the inner and outer cylinders are kept at constant temperatures with the inner surface at a slightly higher temperature than that of the outer. The main objective of this study is to investigate the buoyancy-induced flow as affected by the presence of the porous layer.

Y.D. Zhu and al. [9] presented a natural convective heat transfer between two horizontal, elliptic cylinders that was numerically studied using the differential quadrature (DQ) method. The governing equations are taken to be in the vorticity-stream function formulation. The coordinate transformation was performed to apply the DQ method. An elliptic function was used, which makes the coordinate transformation from the physical domain to the computational domain be set up by an analytical expression. Wassim C. and al. [10] presented a new calculation code using a two-dimensional finite element method valid in a steady and laminar flow within an annular enclosure which is represented by inner circular and outer elliptical cylinders. Mota and al. [11] solved the two-dimensional Darcy-Boussinesq equations, governing natural convection heat transfer in a saturated porous medium, in generalized orthogonal coordinates, using high-order compact finites differences on a very fine grid. The mesh is generated numerically using the orthogonal trajectory method. The code is applied to horizontal eccentric elliptic annuli containing saturated porous media. Charrier-Mojtabi [12] carried a numerical investigation of two-dimensional and three-dimensional free convection flows in a saturated porous horizontal annulus heated from the inner surface, using a Fourier-Galerkin approximation for the periodic azimuthal and axial directions and a collocation-Chebyshev approximation in the confined radial direction. The numerical algorithm integrates the Darcy-Boussinesq's equations formulated in terms of pressure and temperature. M. M. Elshamy and M. N. Ozisik [13] studied numerically a steady-state natural convection for air bounded by two confocal horizontal elliptical cylinders for the case of inner hot and outer cold isothermal surfaces. The local and average Nusselt numbers were determined for different value of Rayleigh number for different eccentricities of the inner surface.

Our interest in considering an annular elliptical geometry is based on their adaptability to become either circular when the axis ratio approaches unity or flat plate when the axis ratio approaches zero. This type of geometry can be found in a wide range of applications such the heat exchangers consisting of coaxial tubes. Our work is studying the heat and mass transfer of natural convection that occurs in the annular space considered as a porous medium saturated with a Newtonian fluid and bounded by two elliptical walls. The effect of Rayleigh-Darcy number on the flow structure, the heat and mass transfer is examined within a range of  $Ra_m = [10-250]$  with buoyancy ratio  $N=1$  and Lewis number  $Le=0.1$ .

## 2. Problem formulation and basic equation

We consider a thermosolutal natural convection in an annular elliptical space filled with fluid-saturated porous medium. Figure 1 represents a cross section of the system. Both elliptic internal and external walls are isothermal and impermeable, kept at constant temperatures and concentrations  $T_1, C_1$  and  $T_2, C_2$  respectively with  $T_1 > T_2$  and  $C_1 > C_2$ . The physical properties of the fluid are constant, apart from the density  $\rho$  whose variations are at the origin of the natural convection.

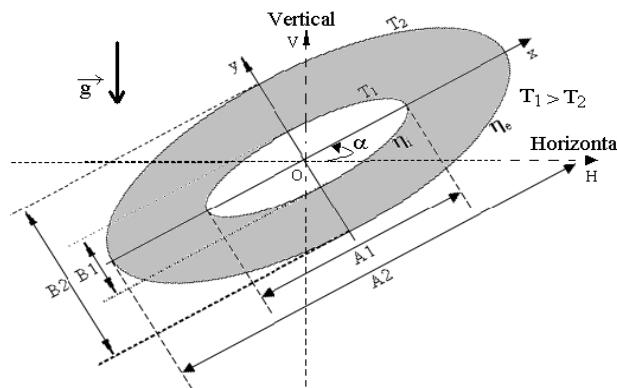


Figure 1: The cross section of the system

Viscous dissipation is neglected, just as the radiation (emissive properties of the two walls being neglected). Soret and Dufour effects are neglected and we admit that the problem is bidimensional, permanent and laminar. The porous medium is considered isotropic and homogeneous. The heat and mass transfer by natural convection is represented by the following equations within the framework of the Boussinesq approximation:

- Continuity equation

$$\text{div} \vec{V} = 0 \quad (1)$$

- Momentum equation

The classic formulation of Darcy is used for writing the equation of motion:

$$\vec{V} = \frac{K}{\mu} (-\nabla P + \rho \vec{g}) \quad (2)$$

- Heat Equation

$$\sigma_T \frac{\partial T}{\partial t} + (\vec{V} \cdot \overrightarrow{\text{grad}}) T = \frac{\lambda_p}{\rho C_p} \nabla^2 T \quad (3)$$

- Concentration Equation

$$\varepsilon \frac{\partial C}{\partial t} + (\vec{V} \cdot \overrightarrow{\text{grad}}) C = D \nabla^2 C \quad (4)$$

The Boussinesq approximation for the combined heat and mass transfer is written as following:

$$\rho = \rho_0 (1 - \beta_T (T - T_0) - \beta_C (C - C_0)) \quad (5)$$

It is convenient to define a reference frame such as the limits of the system result in constant values of the coordinates. The passage from the Cartesian coordinates  $(x, y)$  to the elliptic coordinates  $(\eta, \theta)$  is obtained by the following relations:

$$\begin{cases} x = c \cdot \text{ch}(\eta) \cdot \cos(\theta) \\ y = c \cdot \text{sh}(\eta) \cdot \sin(\theta) \end{cases} \quad (6)$$

The metric coefficients in the elliptic coordinates are given by:

$$h = h_1 = h_2 = c \left( \text{sh}^2(\eta) + \sin^2(\theta) \right)^{1/2}, \quad h_3 = 1$$

$$\text{With } c = \frac{A}{\text{ch}(\eta)} = \frac{B}{\text{sh}(\eta)}$$

Equations (1), (2), (3) and (4) are re-written in the elliptic coordinates using the metric coefficient  $h$  and represented respectively by equations (7), (8), (9) and (10).

$$\frac{\partial}{\partial \eta} (h V_\eta) + \frac{\partial}{\partial \theta} (h V_\theta) = 0 \quad (7)$$

$$\frac{1}{h} \left[ \frac{\partial^2 \psi}{\partial \eta^2} + \frac{\partial^2 \psi}{\partial \theta^2} \right] = - \frac{K g \beta}{\nu} \left( \left[ \cos(\alpha) F(\eta, \theta) - \sin(\alpha) G(\eta, \theta) \right] \left( \frac{\partial T}{\partial \eta} + N \frac{\partial C}{\partial \eta} \right) - \left[ \sin(\alpha) F(\eta, \theta) + \cos(\alpha) G(\eta, \theta) \right] \left( \frac{\partial T}{\partial \theta} + N \frac{\partial C}{\partial \theta} \right) \right) \quad (8)$$

$$V_\eta \frac{\partial T}{\partial \eta} + V_\theta \frac{\partial T}{\partial \theta} = a \left( \frac{1}{h^2} \right) \left[ \frac{\partial^2 T}{\partial \eta^2} + \frac{\partial^2 T}{\partial \theta^2} \right] \quad (9)$$

$$V_\eta \frac{\partial C}{\partial \eta} + V_\theta \frac{\partial C}{\partial \theta} = D \left( \frac{1}{h^2} \right) \left[ \frac{\partial^2 C}{\partial \eta^2} + \frac{\partial^2 C}{\partial \theta^2} \right] \quad (10)$$

$V_\eta$  and  $V_\theta$  are the velocity components in the directions  $\eta$  and  $\theta$ . The coefficients  $F(\eta, \theta)$ ,  $G(\eta, \theta)$  used in (8) are given by:

$$F(\eta, \theta) = \frac{sh(\eta)\cos(\theta)}{(sh^2(\eta) + \sin^2(\theta))^{1/2}}, \quad G(\eta, \theta) = \frac{ch(\eta)\sin(\theta)}{(sh^2(\eta) + \sin^2(\theta))^{1/2}}$$

The characteristic quantities used for the dimensionless problem between the inner and the outer elliptic cylinder are the characteristic temperature and concentration  $\Delta T = T_1 - T_2$ ,  $\Delta C = C_1 - C_2$ . The focal length  $c$  in the elliptic coordinates is the reference length and the thermal diffusivity of the fluid  $a$ , the ratio of the thermal diffusivity and the characteristic length ( $a/c$ ) is the characteristic velocity. The dimensionless mathematical model obtained is expressed by the following equations:

$$\frac{\partial}{\partial \eta} (HV_\eta^+) + \frac{\partial}{\partial \theta} (HV_\theta^+) = 0 \quad (11)$$

$$HV_\eta^+ \frac{\partial T^+}{\partial \eta} + HV_\theta^+ \frac{\partial T^+}{\partial \theta} = \left[ \frac{\partial^2 T^+}{\partial \eta^2} + \frac{\partial^2 T^+}{\partial \theta^2} \right] \quad (12)$$

$$HV_\eta^+ \frac{\partial C^+}{\partial \eta} + HV_\theta^+ \frac{\partial C^+}{\partial \theta} = \frac{1}{Le} \left[ \frac{\partial^2 C^+}{\partial \eta^2} + \frac{\partial^2 C^+}{\partial \theta^2} \right] \quad (13)$$

$$\frac{1}{h} \left[ \frac{\partial^2 \psi^+}{\partial \eta^2} + \frac{\partial^2 \psi^+}{\partial \theta^2} \right] = -Ra_m \cdot H \left( [\cos(\alpha)F(\eta, \theta) - \sin(\alpha)G(\eta, \theta)] \left[ \frac{\partial T^+}{\partial \eta} + N \frac{\partial C^+}{\partial \eta} \right] - [\sin(\alpha)F(\eta, \theta) + \cos(\alpha)G(\eta, \theta)] \left[ \frac{\partial T^+}{\partial \theta} + N \frac{\partial C^+}{\partial \theta} \right] \right) \quad (14)$$

Where  $V_\eta$ ,  $V_\theta$  are the components of the dimensionless velocity defined by:

$$V_\eta^+ = \frac{1}{H} \frac{\partial \psi^+}{\partial \theta}, \quad V_\theta^+ = -\frac{1}{H} \frac{\partial \psi^+}{\partial \eta}$$

$Ra_m$  represents Rayleigh-Darcy number which is defined as:  $Ra_m = Ra \cdot Da$

The boundary conditions are expressed as following:

Hot inner wall with high concentration ( $\eta = \eta_i = \text{cst}$ ):

$$V_\eta^+ = V_\theta^+ = \frac{\partial \psi^+}{\partial \eta} = \frac{\partial \psi^+}{\partial \theta} = 0, \quad T_1^+ = 1, \quad C_1^+ = 1$$

Cold outer wall with low concentration ( $\eta = \eta_e = \text{cst}$ ):

$$V_\eta^+ = V_\theta^+ = \frac{\partial \psi^+}{\partial \eta} = \frac{\partial \psi^+}{\partial \theta} = 0, \quad T_2^+ = 0, \quad C_2^+ = 0$$

### 3. Numerical method

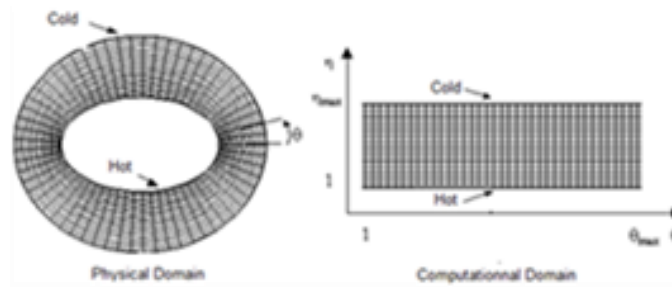


Figure 2: Physical and computational domain

Figure 2 shows the physical and computational domain, to solve (11), (12) and (13) with the associated boundary conditions; we consider a numerical solution by the finite volumes method, exposed by [14]. The power law scheme was used for the discretization. To solve (14), we consider a numerical solution by the centered differences method. The iterative method used for the numerical solution of algebraic system of equations is the Gauss-Seidel with an under-relaxation process. Once the temperature and concentration distributions are available, the local Nusselt and Sherwood numbers in the physical domain are defined as:

$$Nu = -\frac{1}{h} \frac{\partial T^+}{\partial \eta} \Big|_{\eta=cst}$$

$$Sh = -\frac{1}{h} \frac{\partial C^+}{\partial \eta} \Big|_{\eta=cst}$$

## 4. Results and discussion

Our objective is to analyze the effect of Rayleigh-Darcy number for the case of a cooperative mode of the heat and mass transfer. For this reason, we presented streamlines, isotherms and concentration contours for different values of Rayleigh-Darcy number for the case when the buoyancy ratio  $N=1$  and for a determined value of Lewis number  $Le=0.1$ . The Nusselt and Sherwood numbers are presented for different values of  $Ra_m$ . The study was carried out for the case of the air and when the eccentricities of the internal and the external ellipses are respectively given by  $e_1=0.14$  and  $e_2=0.07$  in order to obtain a cylindrical configuration and the inclination of the system is  $\alpha=0^\circ$ .

### 4.1. Influence of Rayleigh-Darcy number ( $Ra_m$ ) on the isolines

Figures 3 to 6 represent the streamlines, isotherms and concentration contours; we note that these contours are symmetrical about the median fictitious vertical plane. The streamlines of the figure 3 show that the flow is organized in two main cells that rotate in opposite directions. This is due to upward movement of the fluid particles under the aiding buoyancy effect related to temperature and solutal gradients, the fluid heat up along the hot wall and the downward movements of the fluid particles which cool along the cold wall under the gravity. Isotherms in figure 3 show that the heat transfer is mainly by conduction in the bottom of the annular space, in the other hand; isotherms deform in the upper space where there is presence of two counter-rotating vortices. This configuration illustrates that the heat transfer is dominated by a convective mode in the upper space, in the lower space the heat transfer is dominated by a conduction mode with a slight contribution of the convection. The concentration contours in this figure are parallel and concentric closed curves which coincide perfectly with the walls profile in the annular space where the mass transfer is purely conducted by diffusion mode

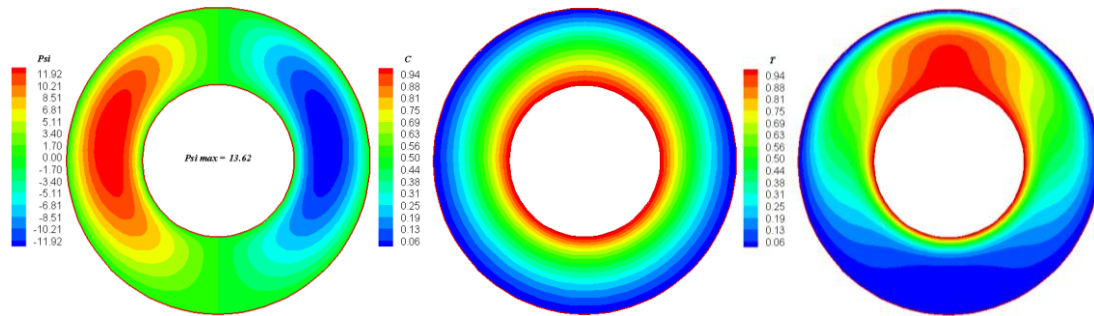


Figure 3: Streamlines, isotherms and concentration contours for  $Ra_m=10$ ,  $N=1$  and  $Le=0.1$

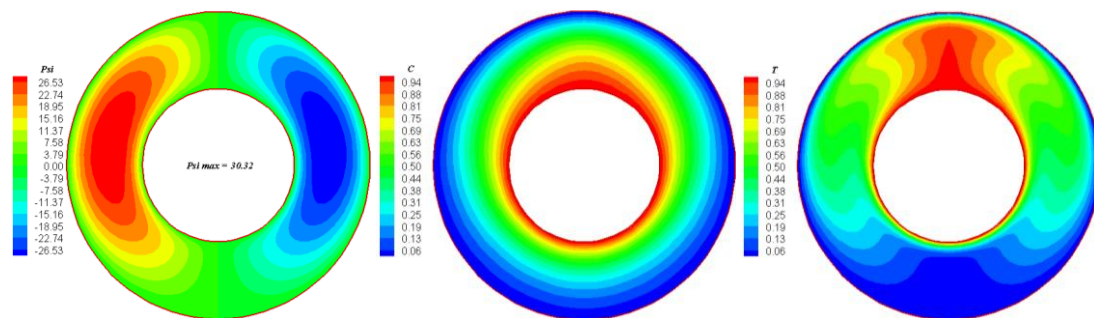


Figure 4: Streamlines, isotherms and concentration contours for  $Ra_m=30$ ,  $N=1$  and  $Le=0.1$

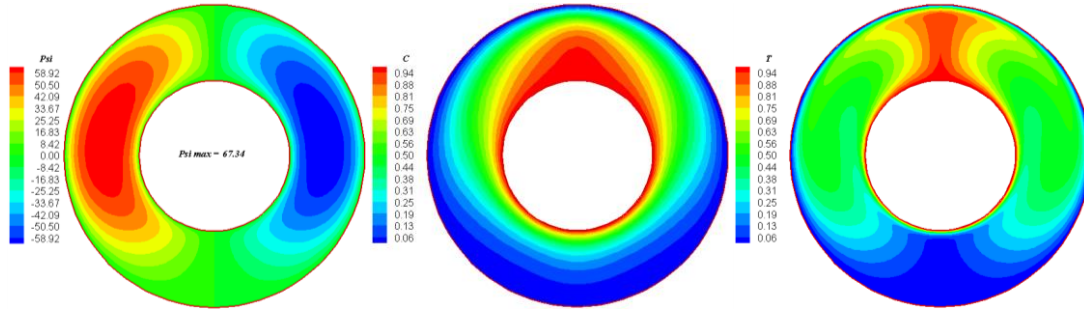


Figure 5: Streamlines, isotherms and concentration contours for  $Ra_m=100$ ,  $N=1$  and  $Le=0.1$

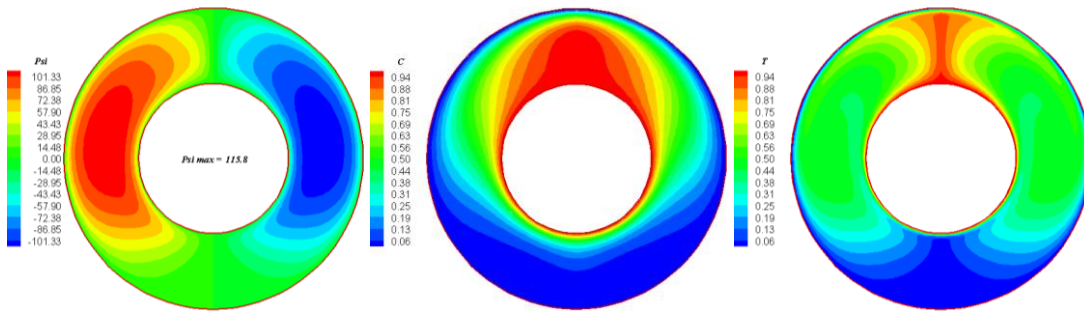


Figure 6: Streamlines, isotherms and concentration contours for  $Ra_m=250$ ,  $N=1$  and  $Le=0.1$

In the figure 4 and 5 we notice that with the increase of Rayleigh-Darcy number and the flow remains organized in two main cells rotating in opposite directions with an increase in the stream function value due to the thermal gradient with increasing Rayleigh-Darcy number. Isotherms show that the convective mode is gaining more space in the bottom section under the effect of thermal gradient. The concentration contours show that the mass transfer is driven by a diffusive mode with a slight transition in the upper space due to the aiding buoyancies. In the figure 5 when the  $Ra_m=100$  streamlines remain organized in two main cells rotating in opposite directions with a decreasing in the upper gap between the rotating cells, the value of stream function is also increasing dramatically with the increase of Rayleigh-Darcy number. Isotherms in the same figure show that the heat is fully distributed by a convective mode in the whole annular space. The concentration contours start to deform in the upper space under the effect of thermal and solutal buoyancies. Figure 6 shows that the streamlines from both cells tend to become adjacent which decrease the gap between the cells in the upper annulus space, the flow remains organized in two main cells rotating in opposite directions with very high motion, this increase is interpreted in the stream function values that are increasing due to the increase of the thermal gradient as consequence of the increase in the Rayleigh-Darcy number. In the same figure, isotherms have a significant change and are increasingly distorted at the top of the annular space due to the increase of the thermal gradient. The concentration contours illustrate a solutal stratification relatively decreasing in the upper section of the annular space, the distortion of the curves in the upper space shows that the mass transfer is conducted by the convective mode which is taking place in the annular.

#### 4.2. The effect of Rayleigh-Darcy number on local Nusselt and Sherwood numbers

In the figure 7 we illustrated the variation of local Nusselt and Sherwood numbers on the inner wall of the cylinder in term of Rayleigh-Darcy number. For the local Nusselt number this variation allows us to note that with the increase of Rayleigh-Darcy number, the value of local Nusselt number increase significantly due to the increase in the thermal gradient which is obvious. The Nusselt variation in figure 7 shows for  $Ra_m=250$  that the maximum value correspond to  $\theta=270^\circ$  where the convection is taking place in the region of the cold fluid at the lower annular space. The local Sherwood number which interprets the mass transfer allows us to note that the

Sherwood number increases with increasing of Rayleigh-Darcy number due to the aiding effect of thermal and solutal buoyancies. The mass transfer is enhanced due the convection mode that take place in the upper annular space, however, the solutal distribution remains dominated by a diffusive mode due to the high solutal diffusivity compared to the thermal diffusivity when  $Le=0.1$  and for this same reason we have Sherwood values very small compared to Nusselt values.

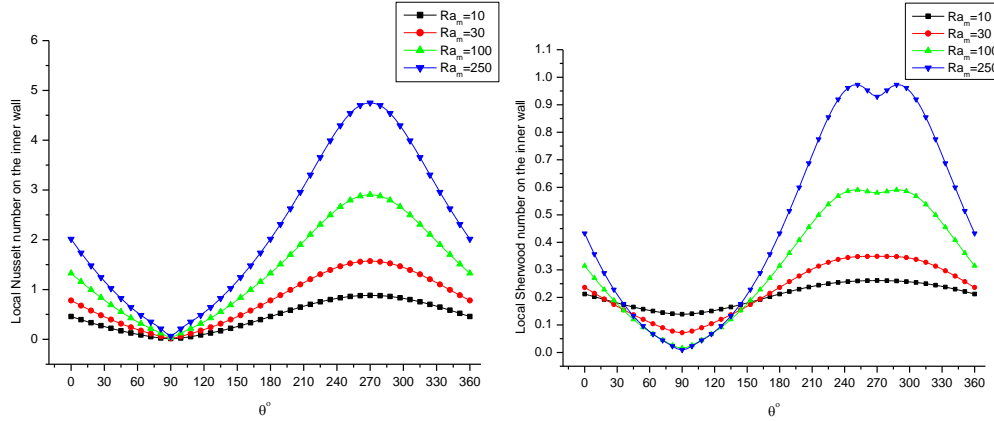


Figure 7: Variation of local Nusselt and Sherwood numbers on the inner wall for  $Le=0.1$  and  $N=1$

## Conclusion

Heat and mass transfer of natural convection in a porous cylindrical annulus saturated by a Newtonian fluid was studied by a numerical method using the method of finite volumes and the vorticity-streamline formulation. We examined, in particular, the influence of Rayleigh-Darcy number in a range of [10-250] for the case when the thermal and the solutal buoyancies are equal and cooperating in the generation of the flow structure. The structures of bicellular convection take place according to the value of the Rayleigh-Darcy number. Both heat and mass transfer distributions are very sensitive to the variation of Rayleigh-Darcy number. When increasing Rayleigh-Darcy number, the heat transfer is dominated by the convective mode in the entire annular space. The mass transfer remains dominated by a diffusive mode due to the high solutal diffusivity which is ten times higher than the thermal diffusivity for  $Le=0.1$ . With the increase of Rayleigh-Darcy number the convection mode of the mass transfer rises in the upper space as consequence of the aiding effect of the thermal and the solutal buoyancies when the buoyancy ratio  $N=1$ .

## Nomenclature

Symbol	Definition	Unit			
$a$	Thermal diffusivity	$m^2/s$	$Gr$	Grashof number,	
$A$	Elliptic cylinder major axis	$m$	$h$	$[=g\beta c^3(T_1-T_2)/\nu^2]$	
$B$	Elliptic cylinder minor axis	$m$	$H$	Dimensional metric coefficient	$m$
$c$	Constant defined in elliptic coordinates	$m$	$K$	Dimensionless metric coefficient	
$c_p$	Specific heat at constant pressure	$J/kg.K$	$Le$	Porosity	$m^2$
$D$	Concentration diffusivity	$m^2/s$	$N$	permeability	
$C$	Mass concentration	$g/l$	$Pr$	Lewis Number	
$C_1$	Inner wall's concentration	$g/l$	$Ra$	Buoyancy ratio,	
$C_2$	Outer wall's concentration	$g/l$	$Ra_m$	$[=\beta c AC/\beta_T AT]$	
$Da$	Darcy number		$Nu$	Local Nusselt number	
$e$	Elliptic cylinder eccentricity,	$[\frac{A^2-B^2}{A^2}]$	$P$	Pressure	$N/m^2$
$g$	Gravitational acceleration	$m/s^2$	$Pr$	Prandtl number	
			$Ra$	Rayleigh number, $[=Gr.Pr]$	
			$Ra_m$	Rayleigh-Darcy number, $[=Ra.Da]$	

$Sh$	Local Sherwood number			coefficient	
$t$	Time	$s$	$\lambda$	Thermal conductivity	$W/m.K$
$T$	Fluid's temperature	$K$	$\nu$	Kinematic viscosity,	$m^2/s$
$T_I$	Inner wall's temperature	$K$	$\varepsilon$	Porosity	
$u$	Velocity component- coordinate x	$m/s$	$\sigma_T$	Thermal capacity factor	
$v$	Velocity component- coordinate y	$m/s$	$\Delta C$	Concentration difference, [ $=C_1-C_2$ ]	$g/l$
$U_\eta$	Velocity component- coordinate $\eta$	$m/s$	$\Delta T$	Temperature difference, [ $=T_1-T_2$ ],	$K$
$V_\theta$	Velocity component- coordinate $\theta$		$\rho$	Density	$kg/m^3$
			$\psi$	Stream function	$m^2/s$
			$\eta, \theta$	Elliptic coordinates	
			<i>Subscripts</i>		
$\vec{v}$	Velocity vector	$m/s$	1	Inner	
$x, y$	Cartesian coordinates	$m$	2	Outer	
			$p$	Porous	
<i>Greek Letters</i>					
$\alpha$	Inclination angle	$^\circ$	<i>Superscripts</i>		
$\beta_C$	Concentration expansion coefficient	$(g/l)^{-1}$	+	Dimensionless parameters	
$\beta_T$	Thermal expansion	$K^{-1}$			

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